

round trip with a fixed-vector wind

saw this one a while back and it took me 25 minutes, which is longer than it should have. i got there with limits, one of the two usual ways in. i would love for you to try it.

a particle goes from A to B and back, along the straight segment $\overline{AB}$ of length d . its speed relative to the air is always v .

the air moves at a constant velocity $\mathbf{w}$, with $|\mathbf{w}| < v$, the same on both legs. it can point in any direction, including across $\overline{AB}$ rather than along it. $\mathbf{w} = \mathbf{0}$ is allowed.

let $T(\mathbf{w})$ be the round trip time. is there a $\mathbf{w}$ with $|\mathbf{w}| < v$ such that

$$T(\mathbf{w}) < T(\mathbf{0})?$$